OPTIMIZING CLAIMS FLUCTUATION RESERVES

by

Christoph Haehling von Lanzenauer[*]
and Don D. Wright[**]

September, 1975   WP 822-75

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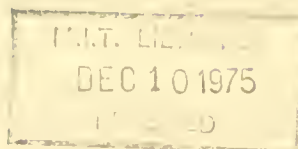Management, M.I.T.

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I. INTRODUCTION

Some thirty years ago group insurance did not exist for all
practical purposes. A number of conditions led to developments
which has made group insurance today a multi-billion dollar
business [7]. Although the principles of insurance are the same,
the effects of individual insurance and group insurance are quite
different. In individual insurance the insured transfers for a
premium the risk of the unknown financial consequences to the
insurer. In group insurance, the financial consequences, although
unknown are much more predictable, allowing a shift of most or
all the risk back to the group of policy holders. The fact
that the majority of risk in group insurance lies with the policy
holders led many employers to seek more efficient ways of spending
their insurance dollars.

Various alternatives have been developed over the years and
include Experience Rated Plans, Minimum Premium Arrangement,
Administrative Services Only (ASO), just to name a few. While
the administrative form may be different, many arrangements are
expected to pay their own way and represent more or less self-
insurance. These developments have added a new dimension to the
area of risk management and employee benefit programs. They require
new and different skills from the risk manager in dealing effective-
ly with issues like assessment of risk, how much self insurance to
accept and how to cope with assumed risks.

The purpose of this paper is to present some problems of a fully
experience rated group life insurance plan and to demonstrate the

demonstrate the contribution Management Science concepts can make in analyzing and solving these problems.

II. PROBLEM

Background

As a result of negotiations between management and the employees of a large organization,[1] a fully experience rated, yearly renewable group life insurance plan was introduced in 1971 and became effective January 1, 1972. The plan provided death benefits of twice the annual salary rounded to the next higher thousand with a minimum of $10,000 and a maximum of $40,000 for the 4,258 eligible employees. The premium per $1,000 coverage was $.34 per month and was paid in equal parts by the organization and its employees. The premium level was based on standard premium rates applicable to a given classification of groups. The plan - being fully experience rated - represented in fact self insurance and was administrated for a retention fee by a large insurance company. At the end of each policy year a surplus or deficit position was determined according to the formula:

$$\left.\begin{array}{l}\text{Surplus} \\ \\ \text{Deficit}\end{array}\right\} = \left\{\begin{array}{l}\text{Premium} \\ \text{-Retention} \\ \text{-Claims Charges} \\ \text{-Premium Tax}\end{array}\right\} \quad\begin{array}{l}\text{if} \geq 0 \\ \\ \\ \text{if} < 0\end{array}$$

with

$$\text{Premium Tax} = .02 \ (\text{Premium-Surplus}).$$

The annual retention fee is given in Exhibit I.

EXHIBIT I

Annual Retention Fee (Renewal Only)

Policy Fee	$ 120
Certificate Charge	5160 + .57 (Number of Lives -300)
Overhead	.263% of Premium
Claims Administration	.675% of Claims Charge
Contingency Charge	1500 + .5% of (Premium - 50,000)

Total	$9,357.42 + .675% of Claims Charge

The risk associated with the plan was therefore entirely with the organization and its employees. A surplus resulting from favorable experience was to be returned to the group of policyholders. In the case of unfavorable experience, the insurance company had to be reimbursed for the resulting deficit. Due to competitive pressures in the industry a deficit up to about 10% of the annual premium volume is typically carried forward in the hope of recapturing it by future surpluses. A deficit in excess of that amount would trigger a premium increase.

Although the decision to self insure through the fully experience rated plan had been made, the problem of how to cope with the volatile nature of life insurance claims was still unresolved. More specifically the interrelated issues of developing a decision rule of how to dispose of any suplus and making a recommendation of what

should be done (if anything) to avoid upward premium adjustments had to be addressed.

Decision

At the beginning of 1974 a surplus of $76,518.00 had been accumulated as a result of favorable experience during the first two years of operation. Due to pressures from the employees, a policy decision regarding the use of the existing and future surpluses had to be made. Various alternatives presented themselves and included:

      (a)   To refund this and any future surplus in form of rebates or premium discounts and to assume the risk of premium increases resulting from deficit positions.

      (b)   To use the surplus to establish and fund a claims fluctuation reserve as a means of coping with the volatile nature of claim charges. Any surplus in excess of the funds required for the reserve would be refunded.

      (c)   To use the surplus to purchase, for example, stop loss insurance which shifts the responsibility of losses in excess of an agreed stop loss level to an insurance carrier.

Purchasing stop loss or other insurance was not considered to be advisable since it was inconsistent with the idea of a fully experience rated plan. Furthermore, any form of insurance would raise the premium level and would result in serious dissatisfaction of the employees.

The purpose of a claims fluctuation reserve is to provide a sink for surplus and a source of funds in case of a deficit. The reserve

absorbs the effects of fluctuations in claims which otherwise
would result in frequent rebates and premium increases. A claims
fluctuation reserve is specified by two design parameters, an
upper level A and a lower level Z.[2] If the accumulated funds in
the reserve exceed A, the excess amount will be redistributed.
The operational problem of determining the rebate in year t
$(t = 1, 2, \ldots)$, B_t $(B_t \geq 0)$, can now be expressed by

$$
(1) \qquad B_t = \begin{cases} Y_t - A & \text{if } Y_t > A \\ \\ 0 & \text{if } Y_t \leq A \end{cases}
$$

with Y_t being the accumulated amount in the reserve at the end
of year t. If, on the other hand, the accumulated deficit is
more than Z, a premium increase will take place indicating that
the existing premium level is inadequate for the exposed risk.
The level Z was determined by competitive pressures in the industry
and set at - 10% of the annual premium volume.

Since alternative (a) is a special case of a claims fluctuation
reserve with A = O, the problem can be defined as selecting the
design parameter A according to a given criterion.

Criterion

Although economic factors were recognized (e.g. opportunity cost
for funds tied up in the reserve, administrative expenses in
connection with premium rebates), it was decided to focus on the
cost of providing coverage and the possibility of upward premium
adjustments in selecting a value for A. Since the premium was a

a most sensitive subject with the employees, rebates should be maximized and premium increase avoided if at all possible. Recognizing that a premium increase is a random variable, the criterion of mimimizing A (i.e. maximizing the rebate) subject to a probabilistic constraint on the time (in years) to the next premium increase T (T = 1,2,...) was selected. Thus

(2)   minimize {A}

subject to

(3) $P(T \leq \tau | A) \leq \alpha$

with τ and α being managerially determined quantities.

The criterion suggested warrants some discussion. While the operational problem of determining the yearly rebate accounts for the existing condition (i.e. the actual amount in the reserve, Y_t) the determination of the design parameter A must be made without regard to Y_t. If the probabilistic constraint (3) would be conditional on Y_t, the design parameter A would not only change as Y_t changes, but A would also be unrealistically large for small values of Y_t. The criterion suggested does not imply that existing conditions are irrelevant. On the contrary, the information contained in

(4) $P(T \leq \tau | Y_t)$

is of key importance in assessing risk and may be used as an argument to generate additional funds for initial financing, but (4) has nothing to do with selecting the design parameter A.

III.   THE UNDERLYING STOCHASTIC PROCESS

To derive the distribution of T, the underlying stochastic process
must be developed.

The key variable in group life insurance plans is the aggregate
claims in a given period, say a year, and represents the total
amount paid in claims resulting from the coverage provided by
the plan.   Let X ($X \geq 0$) be the aggregate claims in period t
and $f_t(X)$ the corresponding density or mass function.   X is
a random variable depending on the number of deaths n (n=0,1,2,...)
which may occur and the amount isured x ($10,000 \leq x \leq 40,000$) of
those who die.   Thus, $f_t(X)$ is a compound distribution.

The derivation of $f_t(X)$ is a central problem in actuarial science
[6], since $f_t(X)$ forms the basis for many risk management problems
such as assessment of risk, premium calculation, determination of
insurance and reinsurance coverage.   In view of the absence of
worthwhile historical data as a basis for "deriving" $f_t(X)$ - each
year produces only one data point -, analytical approaches
have to be taken [1,3,4,5,6].

Let $\pi_t(n)$ be the distribution of the variable number of deaths
n (n=0,1,2,....). $g_t(x)$ denotes the distribution of x
($10,000 \leq x \leq 40,000$) with $G_t(x)$ being the corresponding distri-
bution function.   Assuming independence between deaths, the
distribution function of the sum of n individual claims x is
the nth convolution of $G_t(X)$ which is defined as $G_t^{n*}(X)$ and
can be obtained recursively for n=1,2,... by

(5) $\qquad G_t^{n*}(X) = \int_o^X G_t^{(n-1)*}(X-x)\, g_t(x)\, dx \qquad n \geq 1$

If we define

(6) $\qquad G_t^{o*}(X) = 1 \qquad\qquad n=0 \text{ and } X \geq 0$

the distribution function of X, $F_t(X)$, can be expressed as

(7) $\qquad F_t(X) = \sum_{n=o}^{\infty} \pi_t(n)\, G_t^{n*}(X) \qquad X \geq 0$

Thus, $f_t(X)$ can be written as

(8) $\qquad f_t(X) = \dfrac{dF_t(x)}{dx} = \sum_{n=0}^{\infty} \pi_t(n)\, g_t^{n*}(X).$

The immediate problem is the numerical evaluation of $f_t(X)$.

Explicit results for $f_t(X)$ can only be obtained under restrictive conditions [6]. An effective way of specifying $f_t(X)$ is to determine the mean and higher moments of X and to employ the method of moments to find the parameters of a distribution to represent $f_t(X)$. Details of the derivation of the moments of X are given in [1,3,4] and it is shown that the moments of X can be expressed entirely by $\pi_t(n)$ and the moments of $g_t(x)$. The approach requires therefore only the knowledge of $\pi_t(n)$ and the moments of $g_t(x)$.

$\pi_t(n)$ can be derived from the mortality table used for the respective group. Let q_i^M $[q_i^F]$ represent the probability of death for a male [female] employee of age i during the current year. If we represent the distribution of the number of deaths for the members of age group i by a binomial distribution, we imply that a deceased employee is not replaced during the current year (individual risk theory). A more realistic assumption for a large

organization is to replace an employee who died. The assumption
of collective risk theory is to replace a deceased employee in-
stantly by an individual having identical characteristics.
This allows the possibility of "multiple deaths" and makes the
Poisson distribution the appropriate choice.

Since $1-q_i$ is the probability of no death, we can equate [5]

$$(9) \qquad P(n=0 \mid \lambda_i) = \frac{\lambda_i^0}{0!} e^{-\lambda_i} = 1-q_i$$

or

$$(9b) \qquad e^{-\lambda_i} = 1-q_i$$

leading to

$$(10) \qquad \lambda_i = -\ln(1-q_i).$$

The Poisson parameter λ for the entire group can be derived by

$$(11) \qquad \lambda = \sum_i (\lambda_i^M M_i + \lambda_i^F F_i)$$

with M_i [F_i] representing the number of male [female] employees
of age group i.

The distribution of the amount insured, $g_t(x)$, can be defined by

$$(12) \qquad g_t(x=C_k) = \frac{\sum_i (\lambda_i^M M_i^k + \lambda_i^F F_i^k)}{\sum_k \sum_i (\lambda_i^M M_i^k + \lambda_i^F F_i^k)}$$

with C_k representing the possible amount that can be claimed
(i.e. \$10,000, \$11,000,.....,\$40,000), and M_i^k [F_i^k] the number
of male [female] employees of age i being insured for amount C_k.
The moments of $g_t(x)$ can be easily be determined from (12).

Various distributions can be used to represent $f_t(X)$. Although $f_t(X)$ is a discrete distribution for the problem presented, $f_t(X)$ had been represented by a Beta distribution which can be used effectively in many practical situations [3,4]. A graph of $f_t(X)$ for the problem is given in Exhibit I.

EXHIBIT I

The Distribution of Aggregate Claims - $f_t(X)$
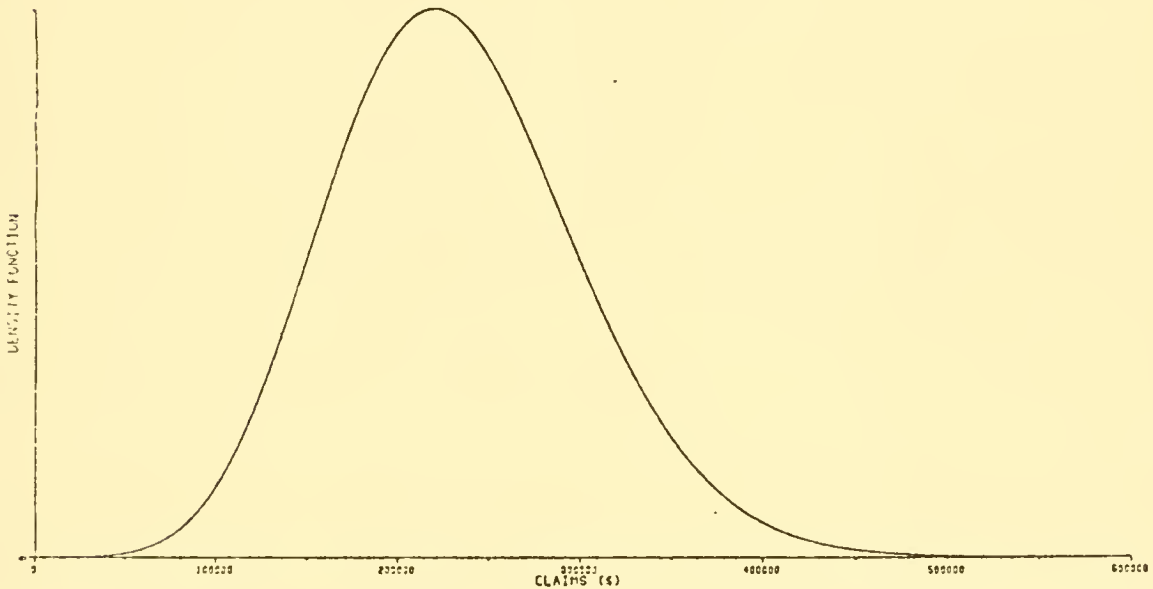
Since only a few of the moments of X are used to fit a distribution, some error may be introduced. An indication of the amount of error can be obtained by comparing the moments of X not utilized to the corresponding moments of the fitted distribution [3].

IV. THE DISTRIBUTION OF T

The amount of funds in the claims fluctuation reserve at the end
of a policy year t, Y_t, will depend on the funds available at
the beginning of the year and the claims experience throughout
the year as determined by $f_t(X)$. A surplus resulting from
favorable experience will increase the funds in the reserve up
to the upper level A, while a deficit reduces the initial amount
with a premium increase being triggered if Y_t falls below 10% of
the annual premium volume. Y_t is therefore a random variable.
Y_t can be discretized by letting the index r (r=1,2.....,R) rep-
resent the possible values of Y_t at time t (state variable) with
r=1 being A and r=R being Z. A transition from state r to state
s is clearly governed by the underlying stochastic process $f_t(X)$.
Let $p_{rs}(t)$ represent the one step transition probability. Thus

(13) $$p_{rs}(t) = p\{Y_{t+1}=s \mid Y_t=r\}.$$

A transition from state $Y_t = r$ to state $Y_{t+1} = s$ occurs if the
aggregate claims X in period t are equal to m as determined by

(14) $$X = \begin{Bmatrix} Y_t \\ +\text{Premium} \\ -\text{Retention} \\ -\text{Premium Tax} \\ -Y_{t+1} \end{Bmatrix} / 1.00675 = m.$$

Thus,

(15) $$p_{rs}(t) = \int_a^b f_t(X) \, dX$$

with $a = m-\Delta/2$ and $b = m+\Delta/2$ assuming an increment size of Δ between states r and $r+1$.

Stationary Conditions

If $f_t(X)$ is stationary, i.e. the size of the group, its age distribution and the distribution of the amount insured remain constant over time, the transition probabilities $p_{rs}(t)$ will be stationary. Thus,

$$(16) \qquad p_{rs}(t) = p_{rs}(t+1) \qquad \text{for all } t.$$

Let $h_{rs}^{(T)}$ represent the probability that the first passage time from state r to state s is $T (T=1,2,....)$ years. Defining $h_{rs}^{(T)} = 0$ for $T \leq 0$, $h_{rs}^{(T)}$ can be obtained recursively by

$$(17) \quad h_{rs}^{(T)} = p_{rs}^{(T)} - h_{rs}^{(1)} p_{ss}^{(T-1)} - h_{rs}^{(2)} p_{ss}^{(T-2)} - .... - h_{rs}^{(T-1)} p_{ss}.$$

The distribution of T, the time to the next premium increase, is therefore the distribution of the first passage time with $r=A$ and $s=Z$. The probabilistic constraint (3) can now be expressed by

$$(18) \qquad P(T \leq \tau | A) = \sum_{T=1}^{\tau} h_{rs}^{(T)}$$

with $r=A$ and $s=Z$.

Non Stationary Conditions

The assumption of stationary conditions is rather weak and should be replaced by the dynamics of a group life insurance plan which may result in changes of the group size and of the age- and amount insured distributions. The approach presented however, can still be applied by simply deriving the distribution of aggregate claims

for each year using the prevailing conditions. Correspondingly the transition probabilities must be derived for each year which are then used for the derivation of the first passage time distribution.

Interest and Tax Considerations

Depending on the specific agreement made between the administrating insurance company and the policy holders, the funds tied up in the reserve may be invested and will earn interest and that income may or may not be subject to tax depending on the specific circumstances.[3] The transition probability $p_{rs}(t)$ must be modified to account for the interest and tax aspects. While interest and tax aspects do not apply when $Y_t \leq 0$, (14) must be changed to (14a) for $Y_t > 0$.

$$(14a) \qquad X_t = \left\{ \begin{array}{l} Y_t[1+\rho][1-d] \\ +\text{Premium} \\ -\text{Retention} \\ -\text{Premium Tax} \\ -Y_{t+1} \end{array} \right\} /1.00675 = m$$

with ρ being the interest rate and d the tax rate.

V. RESULTS

Stationary Conditions

Part of the cummulative distribution of T is given in Exhibit II for various levels of A. For $\tau = 5$ and $\alpha = .10$ the optimal value for A is $140,000, while for $\alpha = .05$ the optimal value of A increases to $180,000. No rebate will be made according to (1) as the accumulated surplus of $76,518 is less than A.

Exhibit II

Distribution of T: Initial Amount = A

τ \ A	$135,000	$140,000	$160,000	$175,000	$180,000
1	.007008	.005952	.003134	.001845	.001539
2	.025611	.022643	.013817	.009362	.008200
3	.049838	.044891	.029497	.021275	.019023
4	.075944	.069159	.047493	.035528	.032150
5	.102325	.093858	.066362	.050849	.046372

With an initial amount in the reserve of only approximately $75,000, the partial distribution of T according to (4) is given in Exhibit III for A being $140,000 and $180,000 respectively.   The results of Exhibit III indicate that with less than full initial funding of the reserve, the chance of a premium increase within five years is considerably larger than .10 or .05 respectively, a risk unavoidable with only partial funding.   This has important implications.   First, it provides information relating to the risk present when the initial amount in the reserve is less than the optimal value A.

Exhibit III

Distribution of T: Initial Amount = $75,000

τ \ A	$140,000	$180,000
1	.042203	.042203
2	.083835	.082737
3	.116205	.111665
4	.143591	.133370
5	.168478	.151132

Second, it may be used as an argument to secure additional funds
for financing the reserve. Finally, this information may be mis-
used by the risk manager to change his critical quantities τ and α.
A reduction of τ for a given value of α will result in a smaller
value for A, whereas a smaller value of α for a given value
of τ increases A. Such adjustments would lead to fluctuating
values of A depending on the funds in the reserve. In determining
the optimal value of A, the amount of funds currently available is
only relevant if in excess of A. It is therefore, of importance
that the person selecting the values for τ and α fully understands
the implications and appreciates the impact of the amount available
for initial funding.

Non Stationary Conditions

Since stationary conditions are not likely to be a good assumption
for what may happen in the next τ years, the analysis has been
carried out for non-stationary conditions.

(a) Increasing Distribution of the Amount Insured

It was assumed that the salaries increase by 10% each year which
has an impact on the amount insured. Typically, the limits on
the coverage are not adjusted frequently. Therefore, we assumed
that $10,000 and $40,000 are the respective values. Exhibit IV
gives part of the distribution of T for various values of A
which indicates that the optimal value for $\tau = 5$ with $\alpha = .10$
and $\alpha = .05$ are $155,000 and $205,000 respectively.

Exhibit IV

Distribution of T: Initial Amount = A

τ \ A	$150,000	$155,000	$200,000	$205,000
1	.004264	.003722	.000758	.000626
2	.019106	.018867	.006100	.005352
3	.041948	.041496	.016605	.014948
4	.071730	.068502	.031240	.028563
5	.103545	.100366	.050350	.046550

As expected the optimal value of A is somewhat larger under such non stationary conditions and the effect of less than full initial funding in the reserve becomes more pronounced (Exhibit V).

Exhibit V

Distribution of T: Initial Amount = $75,000

τ \ A	$155,000	$205,000
1	.036943	.036943
2	.080211	.079380
3	.114032	.109875
4	.143966	.133406
5	.175186	.155249

(b) Increasing Group Size and Distribution of the Amount Insured

Since typically the number of employees grows over time, we assume that the number of policy holders in the group life insurance plan increases by 3% every year. The growth is assumed to materialize in such a fashion that the overall structure of the group in terms of age distribution and male-female split is preserved. (Of course any other assumptions could also be investigated.) The salaries increase as before. As can be observed from Exhibit VI the growth in the group size has only a marginal effect. The optimal value of A is still \$155,000 for α = .10 and \$205,000 for α = .05.

Exhibit VI

Distribution of T: Initial Amount = A

τ \ A	\$150,000	\$155,000	\$200,000	\$205,000
1	.004264	.003722	.000758	.000626
2	.019676	.017421	.005624	.004931
3	.045672	.041445	.016866	.015210
4	.075852	.069801	.032440	.029723
5	.107960	.100276	.050911	.047139

VI. CONCLUSION

The results obtained indicate that no rebate should be made as
the accumulated surplus in less than the upper level A. The
existing surplus should be used for funding the claims fluctuation
reserve. Guidelines are given for selecting the optimal value
for A of the claims fluctuation reserve as a means of coping with
the assumed risk in the fully experience rated group life in-
surance plan. Although the salary increase has a significant
impact on A, for the immediate policy decision regarding rebates,
the stationary assumption is more than adequate. The results
indicate that the dynamics of the problem require periodic review.

The methodology suggested is useful for analyzing and solving the
interrelated problems of determining the annual rebate and selecting
the upper level A of the claims fluctuation reserve. Furthermore,
the approach provides assessment of risk under various conditions.

The paper demonstrates the use of Management Science concepts in
the area of risk management under the current trend towards more
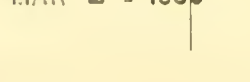self insurance.

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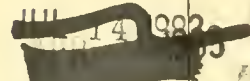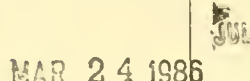
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FOOTNOTES

1) The name of the organization cannot be disclosed, but all
   information, although disguised, represent the setting
   of the actual case.

2) Note the analogy of a claims fluctuation reserve to a safety
   inventory situation.

3) These aspects are of interest for 501 (c)(9) trusts allowed
   by the Internal Revenue Code.